

Adding and Subtracting Integers**Key Vocabulary**additive inverse (*inverso*
aditivo)**ESSENTIAL QUESTION**

How can you use addition and subtraction of integers to solve real-world problems?

EXAMPLE 1**Add.**

A. $-8 + (-7)$ *The signs of both integers are the same.*

$8 + 7 = 15$ *Find the sum of the absolute values.*

$-8 + (-7) = -15$ *Use the sign of integers to write the sum.*

B. $-5 + 11$ *The signs of the integers are different.*

$|11| - |-5| = 6$ *Greater absolute value - lesser absolute value.*

$-5 + 11 = 6$ *11 has the greater absolute value, so the sum is positive.*

EXAMPLE 2**The temperature Tuesday afternoon was 3°C . Tuesday night, the temperature was -6°C . Find the change in temperature.**Find the difference $-6 - 3$.Rewrite as $-6 + (-3)$. *-3 is the opposite of 3 .*

$-6 + (-3) = -9$

The temperature decreased 9°C .**EXERCISES****Add.** (Lessons 1.1, 1.2)

1. $-10 + (-5)$ _____

2. $9 + (-20)$ _____

3. $-13 + 32$ _____

Subtract. (Lesson 1.3)

4. $-12 - 5$ _____

5. $25 - (-4)$ _____

6. $-3 - (-40)$ _____

7. Antoine has \$13 in his checking account. He buys some school supplies and ends up with \$5 in his account. What was the overall change in Antoine's account? (Lesson 1.4)
- _____

Multiplying and Dividing Integers



ESSENTIAL QUESTION

How can you use multiplication and division of integers to solve real-world problems?

EXAMPLE 1

Multiply.

A. $(13)(-3)$

Find the sign of the product. The numbers have different signs, so the product will be negative. Multiply the absolute values. Assign the correct sign to the product.

$$13(-3) = -39$$

B. $(-5)(-8)$

Find the sign of the product. The numbers have the same sign, so the product will be positive. Multiply the absolute values. Assign the correct sign to the product.

$$(-5)(-8) = 40$$

EXAMPLE 2

Christine received -25 points on her exam for 5 wrong answers. How many points did Christine receive for each wrong answer?

Divide -25 by 5.

$$-25 \div 5 = -5$$

*The signs are different.
The quotient is negative.*

Christine received -5 points for each wrong answer.

EXAMPLE 3

Simplify: $15 + (-3) \times 8$

$$15 + (-24) \quad \text{Multiply first.}$$

$$-9 \quad \text{Add.}$$

EXERCISES

Multiply or divide. (Lessons 2.1, 2.2)

1. $-9 \times (-5)$ _____ 2. $0 \times (-10)$ _____ 3. $12 \times (-4)$ _____

4. $-32 \div 8$ _____ 5. $-9 \div (-1)$ _____ 6. $-56 \div 8$ _____

Simplify. (Lesson 2.3)

7. $-14 \div 2 - 3$ _____ 8. $8 + (-20) \times 3$ _____ 9. $36 \div (-6) \times -15$ _____

10. Tony bought 3 packs of pencils for \$4 each and a pencil box for \$7. Mario bought 4 binders for \$6 each and used a coupon for \$6 off. Write and evaluate expressions to find who spent more money.

(Lesson 2.3)

11. Sumaya is reading a book with 288 pages. She has already read 90 pages. She plans to read 20 more pages each day until she finishes the book. Determine how many days Sumaya will need to finish the book. In your answer, count part of a day as a full day. Show that your answer is reasonable.
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MODULE 3 Rational Numbers

Key Vocabulary

rational number (*número racional*)

repeating decimal (*decimal periódico*)

terminating decimal (*decimal finito*)



ESSENTIAL QUESTION

How can you use rational numbers to solve real-world problems?

EXAMPLE 1

Eddie walked $1\frac{2}{3}$ miles on a hiking trail. Write $1\frac{2}{3}$ as a decimal. Use the decimal to classify $1\frac{2}{3}$ according to the number group(s) to which it belongs.

$$1\frac{2}{3} = \frac{5}{3}$$

Write $1\frac{2}{3}$ as an improper fraction.

$$\begin{array}{r} 1.66 \\ 3 \overline{)5.00} \\ \underline{-3} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

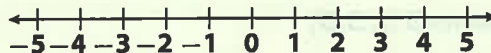
Divide the numerator by the denominator.

The decimal equivalent of $1\frac{2}{3}$ is $1.66\dots$, or $1.\overline{6}$. It is a repeating decimal, and therefore can be classified as a rational number.

EXAMPLE 2

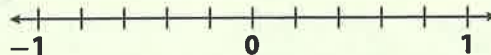
Find each sum or difference.

A. $-2 + 4.5$



Start at -2 and move 4.5 units to the right: $-2 + 4.5 = 2.5$.

B. $-\frac{2}{5} - (-\frac{4}{5})$



Start at $-\frac{2}{5}$. Move $|\frac{4}{5}| = \frac{4}{5}$ unit to the right because you are subtracting a negative number: $-\frac{2}{5} - (-\frac{4}{5}) = \frac{2}{5}$.

EXAMPLE 3

Find the product: $3\left(-\frac{1}{6}\right)\left(-\frac{2}{5}\right)$.

$$3\left(-\frac{1}{6}\right) = -\frac{1}{2}$$

Find the product of the first two factors. One is positive and one is negative, so the product is negative.

$$-\frac{1}{2}\left(-\frac{2}{5}\right) = \frac{1}{5}$$

Multiply the result by the third factor. Both are negative, so the product is positive.

$$3\left(-\frac{1}{6}\right)\left(-\frac{2}{5}\right) = \frac{1}{5}$$

EXAMPLE 4

Find the quotient: $\frac{15.2}{-2}$.

$$\frac{15.2}{-2} = -7.6$$

The quotient is negative because the signs are different.

EXAMPLE 5

A lake's level dropped an average of $3\frac{4}{5}$ inches per day for 21 days. A heavy rain then raised the level 8.25 feet, after which it dropped $9\frac{1}{2}$ inches per day for 4 days. Jayden says that overall, the lake level changed about $-1\frac{1}{2}$ feet. Is this answer reasonable?

Yes; the lake drops about 4 inches, or $\frac{1}{3}$ foot, per day for 21 days, rises about 8 feet, then falls about $\frac{3}{4}$ foot for 4 days:

$$-\frac{1}{3}(21) + 8 - \frac{3}{4}(4) = -7 + 8 - 3 = -2 \text{ feet.}$$

EXERCISES

Write each mixed number as a whole number or decimal. Classify each number according to the group(s) to which it belongs: rational numbers, integers, or whole numbers. (Lesson 3.1)

1. $\frac{3}{4}$ _____

2. $\frac{8}{2}$ _____

3. $\frac{11}{3}$ _____

4. $\frac{5}{2}$ _____

Find each sum or difference. (Lessons 3.2, 3.3)

5. $-5 + 9.5$ _____

6. $\frac{1}{6} + \left(-\frac{5}{6}\right)$ _____

7. $-0.5 + (-8.5)$ _____

8. $-3 - (-8)$ _____

9. $5.6 - (-3.1)$ _____

10. $3\frac{1}{2} - 2\frac{1}{4}$ _____

- 11.** Jorge records his hours each day on a time sheet. Last week when he was ill, his time sheet was incomplete. If Jorge worked a total of 30.5 hours last week, how many hours are missing? Show your work. Then show that your answer is reasonable.

Mon	Tues	Wed	Thurs	Fri
8	$7\frac{1}{4}$	$8\frac{1}{2}$		

Find each product or quotient. (Lessons 3.4, 3.5)

12. $-9 \times (-5)$ _____ **13.** $0 \times (-7)$ _____ **14.** -8×8 _____

15. $\frac{-56}{8}$ _____ **16.** $\frac{-130}{-5}$ _____ **17.** $\frac{34.5}{1.5}$ _____

18. $-\frac{2}{5}(-\frac{1}{2})(-\frac{5}{6})$ _____ **19.** $(\frac{1}{5})(-\frac{5}{7})(\frac{3}{4})$ _____

- 20.** Lei withdrew \$50 from her bank account every day for a week. What was the change in her account in that week?

- 21.** Dan is cutting 4.75-foot lengths of twine from a 240-foot spool of twine. He needs to cut 42 lengths and says that 40.5 feet of twine will remain. Show that this is reasonable.

- 22.** Jackson works as a veterinary technician and earns \$12.20 per hour.

- a.** Jackson normally works 40 hours a week. In a normal week, what is his total pay before taxes and other deductions?

- b.** Last week, Jackson was ill and missed some work. His total pay before deductions was \$372.10. Write and solve an equation to find the number of hours Jackson worked.

- 23.** When Jackson works more than 40 hours in a week, he earns 1.5 times his normal hourly rate for each of the extra hours. Jackson worked 43 hours one week. What was his total pay before deductions? Justify your answer.

Unit Project

COMMON CORE

7.NS.1, 7.NS.2

It's Okay to Be Negative!

In 2012, film director James Cameron piloted a craft called the *Deepsea Challenger* to the deepest point in the Pacific Ocean, 6.8 miles below sea level. The descent took 2.6 hours.

You can use this information and operations with negative rational numbers to find the average rate of descent.

$$\text{rate} = \frac{\text{distance}}{\text{time}} = \frac{-6.8 \text{ mi}}{2.6 \text{ h}} \approx -2.62 \text{ mi/h}$$

For this project, you can use any source in which you can find real-world data involving negative rational numbers. Create a presentation of four original real-world math problems involving negative rational numbers, including their solutions.

You should have one problem for each operation: addition, subtraction, multiplication, and division. Your presentation should include the sources for the information that you used. Use the space below to write down any questions you have or important information from your teacher.



MATH IN CAREERS ACTIVITY

Urban Planner Armand is an urban planner, and he has proposed a site for a new town library. The site is between city hall and the post office on Main Street.



The distance between city hall and the post office is 6.5 miles. City hall is 1.25 miles closer to the library site than it is to the post office. Determine the distance from city hall to the library site and the distance from the post office to the library site.

Study Guide Review

MODULE 4 Ratios and Proportionality

ESSENTIAL QUESTION

How can you use ratios and proportionality to solve real-world problems?

EXAMPLE

A store sells onions by the pound. Is the relationship between the cost of an amount of onions and the number of pounds proportional? If so, write an equation for the relationship, and represent the relationship on a graph.

Number of pounds	2	5	6
Cost (\$)	3.00	7.50	9.00

Write the rates.

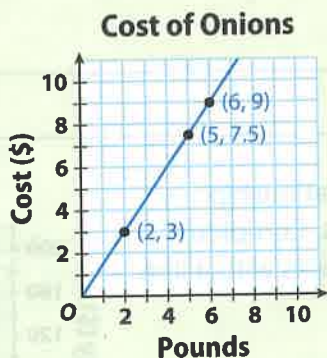
$$\frac{\text{cost}}{\text{number of pounds}} : \frac{\$3.00}{2 \text{ pounds}} = \frac{\$1.50}{1 \text{ pound}}$$

$$\frac{\$7.50}{5 \text{ pounds}} = \frac{\$1.50}{1 \text{ pound}}$$

$$\frac{\$9.00}{6 \text{ pounds}} = \frac{\$1.50}{1 \text{ pound}}$$

The rates are constant, so the relationship is proportional.

The constant rate of change is \$1.50 per pound, so the constant of proportionality is 1.5. Let x represent the number of pounds and y represent the cost.



The equation for the relationship is $y = 1.5x$.

Plot the ordered pairs (pounds, cost): (2, 3), (5, 7.5), and (6, 9).

Connect the points with a line.

Key Vocabulary

constant of proportionality

(*constante de proporcionalidad*)

proportion (*proporción*)

proportional relationship

(*relación proporcional*)

rate of change (*tasa de cambio*)

unit rate (*tasa unitaria*)

EXERCISES

- Steve uses $\frac{8}{9}$ gallon of paint to paint 4 identical birdhouses. How many gallons of paint does he use for each birdhouse? (Lesson 4.1) _____
- Ron walks 0.5 mile on the track in 10 minutes. Stevie walks 0.25 mile on the track in 6 minutes. Find the unit rate for each walker in miles per hour. Who is the faster walker? (Lesson 4.1)

- The table below shows how far several animals can travel at their maximum speeds in a given time. Write each animal's speed as a unit rate in feet per second. Which animal has the fastest speed? (Lesson 4.1)

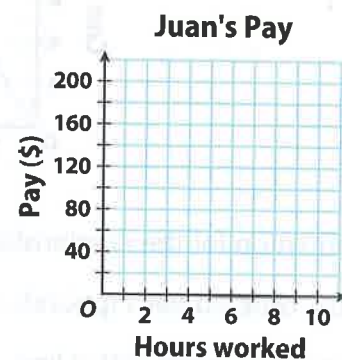
Animal Distances		
Animal	Distance traveled (ft)	Time (s)
elk	33	$\frac{1}{2}$
giraffe	115	$2\frac{1}{2}$
zebra	117	2

- How many miles could the fastest animal travel in 2 hours if it maintained the speed you calculated in exercise 3? Use the formula $d = rt$ and round your answer to the nearest tenth of a mile. Show your work. (Lesson 4.1)

- The data in the table represents how fast each animal can travel at its maximum speed. Is it reasonable to expect the animal from exercise 3 to travel that distance in 2 hours? Explain why or why not. (Lesson 4.1)

- The table below shows the proportional relationship between Juan's pay and the hours he works. Complete the table. Plot the data and connect the points with a line. (Lessons 4.2, 4.3)

Hours worked	2		5	6
Pay (\$)	40	80		



MODULE 5 Proportions and Percent

ESSENTIAL QUESTION

How can you use proportions and percent to solve real-world problems?

EXAMPLE 1

Donata had a 25-minute commute from home to work. Her company moved, and now her commute to work is 33 minutes long. Does this situation represent an increase or a decrease? Find the percent increase or decrease in her commute to work.

This situation represents an increase. Find the percent increase.

amount of change = greater value – lesser value

$$33 - 25 = 8$$

$$\text{percent increase} = \frac{\text{amount of change}}{\text{original amount}}$$

$$\frac{8}{25} = 0.32 = 32\%$$

Donata's commute increased by 32%.

Key Vocabulary

percent decrease

(porcentaje de disminución)

percent increase *(porcentaje de aumento)*

principal *(capital)*

simple interest *(interés simple)*

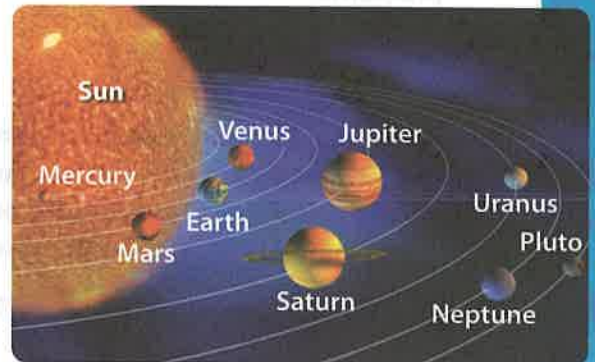
EXERCISES

- Michelle purchased 25 audio files in January. In February she purchased 40 audio files. Find the percent increase in the number of audio files purchased per month. *(Lesson 5.1)* _____
- Sam's dog weighs 72 pounds. The vet suggests that for the dog's health, its weight should decrease by 12.5 percent. According to the vet, what is a healthy weight for the dog? *(Lesson 5.1)* _____
- The original price of a barbecue grill is \$79.50. The grill is marked down 15%. What is the sale price of the grill? *(Lesson 5.2)* _____
- A sporting goods store marks up the cost of soccer balls by 250%. Write an expression that represents the retail cost of the soccer balls. The store buys soccer balls for \$5.00 each. What is the retail price of the soccer balls? *(Lesson 5.2)* _____

To Infinity (Almost)...and Beyond!

For a science project, Orlando decided to make a scale model of the solar system using a scale of 1 inch = 10,000 miles. That would make Earth a sphere about the size of a golf ball.

He quickly discovered that he would need a lot more space than the school could possibly give him. Create a presentation showing the scaled-down sizes and distances that Orlando would need to use for his model solar system. Your presentation should include each of the following:



- The scaled-down diameters, in inches, of the Sun and the planets Mercury, Venus, Mars, Jupiter, Saturn, Uranus, and Neptune, based on a scale of 1 inch = 10,000 miles
- The scaled-down distances from the Sun, in inches, of Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune, based on a scale of 1 inch = 10,000 miles. Base your calculations on the average distances of the planets from the Sun.

To make the scaled-down distances from the Sun easier to visualize, you should convert those of Mercury, Venus, Earth, and Mars to feet and the rest to miles. Use the space below to write down any questions you have or important information from your teacher.

MATH IN CAREERS ACTIVITY

Architect Edith is an architect. She is currently creating a plan to renovate an old warehouse to house a new fitness center. One wall of the warehouse is 36 feet long. Edith plans to increase the length of that wall by 25%. She wants to ensure that there is a minimum of 1 electrical outlet for every 12 feet of length along the wall. What is the length of the new wall? What is the least number of outlets she should include in her plan for that wall? Explain your answer.

MODULE **6** Expressions and Equations**Key Vocabulary**

algebraic expression

*(expresión algebraica)*equation *(ecuación)***ESSENTIAL QUESTION**

How can you use equations to solve real-world problems?

EXAMPLE 1

Huang and Belita both repair computers. Huang makes \$50 a day plus \$25 per repair. Belita makes \$20 a day plus \$35 per repair. Write an expression for Huang and Belita's total daily earnings if they make the same number of repairs r .

$$\text{Huang: } \$50 + \$25r \qquad \text{Belita: } \$20 + \$35r$$

$$\text{Together: } (50 + 25r) + (20 + 35r) = 50 + 20 + 25r + 35r = 70 + 60r$$

Huang and Belita earn \$70 + \$60r together.

EXAMPLE 2

Simplify the expression.

$$1.2(3m - 5) - 7m$$

$$1.2(3m - 5) - 7m$$

$$3.6m - 6 - 7m$$

Use the Distributive Property.

$$3.6m + (-6) + (-7m)$$

Rewrite subtraction as adding the opposite.

$$3.6m + (-7m) + (-6)$$

Use the Commutative Property.

$$-3.4m - 6$$

Combine like terms.

EXAMPLE 3

A skydiver's parachute opens at a height of 2,790 feet. He then falls at a rate of $-15\frac{1}{2}$ feet per second. How long will it take the skydiver to reach the ground?

Let x represent the number of seconds it takes to reach the ground.

$$-15\frac{1}{2}x = -2,790$$

$$-\frac{31}{2}x = -2,790$$

Write as a fraction.

$$\left(-\frac{2}{31}\right)\left(-\frac{31}{2}x\right) = \left(-\frac{2}{31}\right)(-2,790)$$

Multiply both sides by the reciprocal.

$$x = 180$$

It takes 180 seconds for the skydiver to reach the ground.

EXAMPLE 4

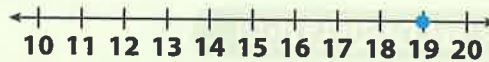
A clothing store sells clothing for 2 times the wholesale cost plus \$10. The store sells a pair of pants for \$48. How much did the store pay for the pants? Represent the solution on a number line.

Let w represent the wholesale cost of the pants, or the price paid by the store.

$$2w + 10 = 48$$

$$2w = 38 \quad \text{Subtract 10 from both sides.}$$

$$w = 19 \quad \text{Divide both sides by 2.}$$



The store paid \$19 for the pants.

EXERCISES

Simplify each expression. (Lesson 6.1)

1. $(2x + 3\frac{2}{5}) + (5x - \frac{4}{5})$ _____

2. $(-0.5x - 4) - (1.5x + 2.3)$ _____

3. $9(3t + 4b)$ _____

4. $0.7(5a - 13p)$ _____

Factor each expression. (Lesson 6.1)

5. $8x + 56$ _____

6. $3x + 57$ _____

Use inverse operations to solve each equation. (Lesson 6.2)

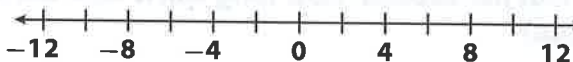
7. $1.6 + y = -7.3$ _____

8. $-\frac{2}{3}n = 12$ _____

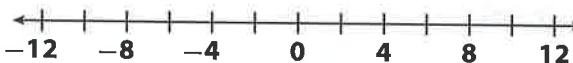
9. The cost of a ticket to an amusement park is \$42 per person. For groups of up to 8 people, the cost per ticket decreases by \$3 for each person in the group. Marcos's ticket cost \$30. Write and solve an equation to find the number of people in Marcos's group. (Lessons 6.3, 6.4)

Solve each equation. Graph the solution on a number line. (Lesson 6.4)

10. $8x - 28 = 44$



11. $-5z + 4 = 34$





ESSENTIAL QUESTION

How can you use inequalities to solve real-world problems?

EXAMPLE 1

Amy is having her birthday party at a roller skating rink. The rink charges a fee of \$50 plus \$8 per person. If Amy wants to spend at most \$170 for the party at the rink, how many people can she invite to her party?

Let p represent the number of people skating at the party.

$$50 + 8p \leq 170$$

$$8p \leq 120 \quad \text{Subtract 50 from both sides.}$$

$$\frac{8p}{8} \leq \frac{120}{8} \quad \text{Divide both sides by 8.}$$

$$p \leq 15$$

Up to 15 people can skate, so Amy can invite up to 14 people to her party.

EXAMPLE 2

Determine which, if any, of these values makes the inequality

$$-7x + 42 \leq 28 \text{ true: } x = -1, x = 2, x = 5.$$

$$-7(-1) + 42 \leq 28$$

$$-7(2) + 42 \leq 28$$

$$-7(5) + 42 \leq 28$$

$$x = 2 \text{ and } x = 5$$

Substitute each value for x in the inequality and evaluate the expression to see if a true inequality results.

EXERCISES

- Prudie needs \$90 or more to be able to take her family out to dinner. She has already saved \$30 and wants to take her family out to eat in 4 days. (Lesson 7.2)

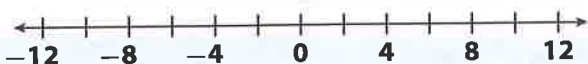
- Suppose that Prudie earns the same each day. Write an inequality to find how much she needs to earn each day.

- Suppose that Prudie earns \$18 each day. Will she have enough money to take her family to dinner in 4 days? Explain.

Solve each inequality. Graph and check the solution. (Lesson 7.3)

2. $11 - 5y < -19$

3. $7x - 2 \leq 61$



The Rhind Papyrus

More than 3,600 years ago, an Egyptian scribe named Ahmose wrote this problem on a type of paper called papyrus: "A quantity with $\frac{1}{7}$ of it added to it becomes 19. What is the quantity?" The problem is Number 24 of 84 problems that are preserved in an ancient manuscript called the Rhind Papyrus. Today we would solve it by writing and solving this equation: $x + \frac{1}{7}x = 19$.

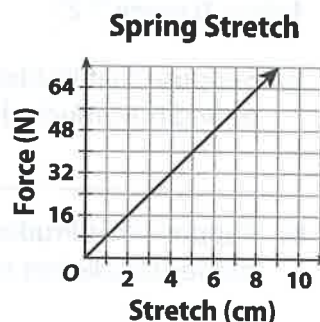


For this project, use the Internet to create a presentation in which you write and solve Problems 24–30, 32, and 34 of the Rhind Papyrus. Express your answers as fractions or mixed numbers.

You should also report on the papyrus itself, giving interesting details about its creation and its discovery in modern times. You can find the problems with a search engine. Use the space below to write down any questions you have or important information from your teacher.

MATH IN CAREERS ACTIVITY

Mechanical Engineer A mechanical engineer is testing the amount of force needed to make a spring stretch by a given amount. The force y is measured in units called *Newtons*, abbreviated N. The stretch x is measured in centimeters. Her results are shown in the graph. Write an equation for the line. Next, explain how you know the graph is proportional. Then, identify the rate of change and the constant of proportionality. Finally, what is the meaning of the constant of proportionality in the context of the problem?



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Study Guide Review

MODULE 8 Modeling Geometric Figures

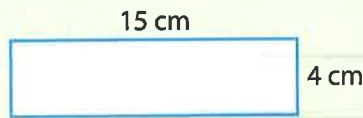


ESSENTIAL QUESTION

How can you apply geometry concepts to solve real-world problems?

EXAMPLE 1

Use the scale drawing to find the perimeter of Tim's yard.



$$\frac{2 \text{ cm}}{14 \text{ ft}} = \frac{1 \text{ cm}}{7 \text{ ft}}$$

1 cm in the drawing equals 7 feet in the actual yard.

$$\frac{1 \text{ cm} \times 15}{7 \text{ ft} \times 15} = \frac{15 \text{ cm}}{105 \text{ ft}}$$

15 cm in the drawing equals 105 feet in the actual yard. Tim's yard is 105 feet long.

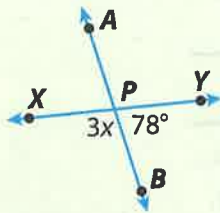
$$\frac{1 \text{ cm} \times 4}{7 \text{ ft} \times 4} = \frac{4 \text{ cm}}{28 \text{ ft}}$$

4 cm in the drawing equals 7 feet in the actual yard. Tim's yard is 28 feet wide.

Perimeter is twice the sum of the length and the width. So the perimeter of Tim's yard is $2(105 + 28) = 2(133)$, or 266 feet.

EXAMPLE 2

Find (a) the value of x and (b) the measure of $\angle APY$.



a. $\angle XPB$ and $\angle YPB$ are supplementary.

$$3x + 78^\circ = 180^\circ$$

$$3x = 102^\circ$$

$$x = 34^\circ$$

b. $\angle APY$ and $\angle XPB$ are vertical angles.

$$m\angle APY = m\angle XPB = 3x = 102^\circ$$

Key Vocabulary

adjacent angles (*ángulos adyacentes*)

complementary angles (*ángulos complementarios*)

congruent angles (*ángulos congruentes*)

cross section (*sección transversal*)

intersection (*intersección*)

plane (*plano*)

scale (*escala*)

scale drawing (*dibujo a escala*)

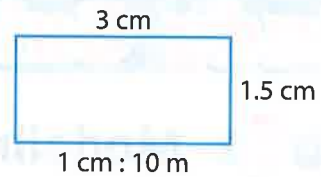
supplementary angles (*ángulos suplementarios*)

vertical angles (*ángulos opuestos por el vértice*)

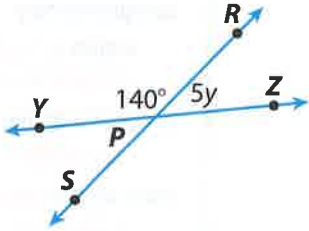
EXERCISES

1. In the scale drawing of a park, the scale is 1 cm : 10 m. Find the area of the actual park.

(Lesson 8.1) _____



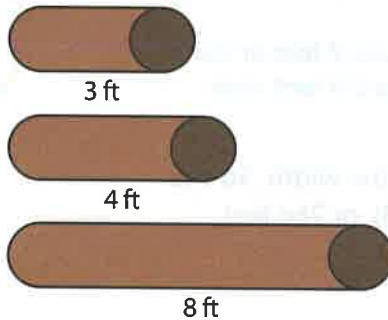
2. Find the value of y and the measure of $\angle YPS$ (Lesson 8.4)



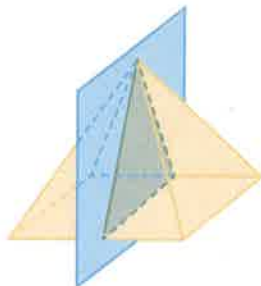
$y =$ _____

$m \angle YPS =$ _____

3. Kanye wants to make a triangular flower bed using logs with the lengths shown below to form the border. Can Kanye form a triangle with the logs without cutting any of them? Explain. (Lesson 8.2)



4. In shop class, Adriana makes a pyramid with a 4-inch square base and a height of 6 inches. She then cuts the pyramid vertically in half as shown. What is the area of each cut surface? (Lesson 8.3)





ESSENTIAL QUESTION

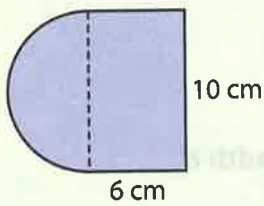
How can you use geometry concepts to solve real-world problems?

Key Vocabulary

- circumference
(circunferencia)
- composite figure *(figura compuesta)*
- diameter *(diámetro)*
- radius *(radio)*

EXAMPLE 1

Find the area of the composite figure. It consists of a semicircle and a rectangle.



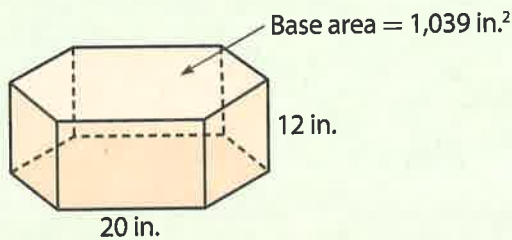
$$\begin{aligned} \text{Area of semicircle} &= 0.5(\pi r^2) \\ &\approx 0.5(3.14)25 \\ &\approx 39.25 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of rectangle} &= \ell w \\ &= 10(6) \\ &= 60 \text{ cm}^2 \end{aligned}$$

The area of the composite figure is approximately 99.25 square centimeters.

EXAMPLE 2

Find the volume and surface area of the regular hexagonal prism hat box shown. Each side of the hexagonal base is 20 inches.



Use the formulas for volume and surface area of a prism.

$$V = Bh$$

$$= 1,039(12)$$

$$= 12,468 \text{ in}^3$$

$$S = Ph + 2B$$

$$= 120(12) + 2(1,039)$$

$$= 1,440 + 2,078$$

$$= 3,518 \text{ in}^2$$

$$\text{Perimeter} = 6(20) = 120 \text{ in.}$$

EXERCISES

Find the circumference and area of each circle. Round to the nearest hundredth. (Lessons 9.1, 9.2)

1.

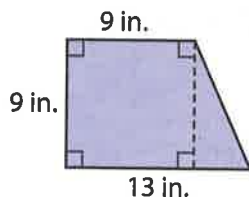


2.



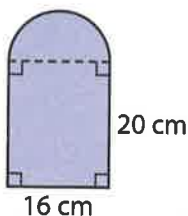
Find the area of each composite figure. Round to the nearest hundredth if necessary. (Lesson 9.3)

3.



Area _____

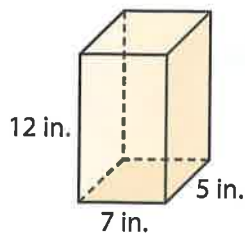
4.



Area _____

Find the volume of each figure. (Lesson 9.5)

5.

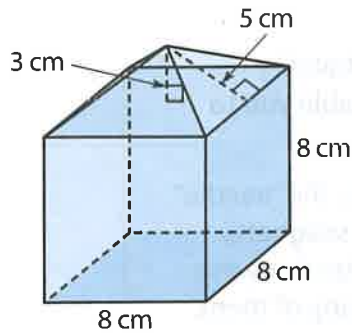


6. The volume of a triangular prism is 264 cubic feet. The area of a base of the prism is 48 square feet. Find the height of the prism.

(Lesson 9.5) _____

EXERCISES

A glass paperweight has a composite shape: a square pyramid fitting exactly on top of an 8-centimeter cube. The pyramid has a height of 3 cm. Each triangular face has a height of 5 centimeters. (Lessons 9.4, 9.5)

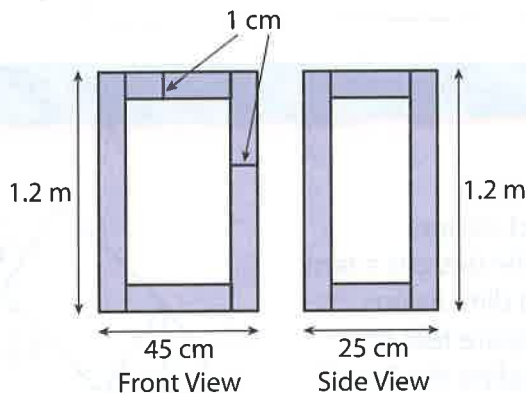


7. What is the volume of the paperweight? _____
8. What is the total surface area of the paperweight? _____

Li is making a stand to display a sculpture made in art class. The stand is a rectangular prism and will be 45 centimeters wide, 25 centimeters long, and 1.2 meters high.

9. What is the volume of the stand? Write your answer in cubic centimeters. (Lesson 9.5)

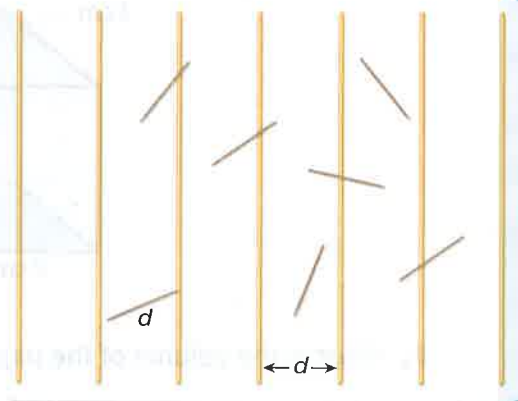
10. Li needs to fill the stand with sand so that it is heavy and stable. Each piece of wood is 1 centimeter thick. The boards are put together as shown in the figure, which is not drawn to scale. How many cubic centimeters of sand does she need to fill the stand? Explain how you found your answer. (Lesson 9.5)



Buffon's Needle

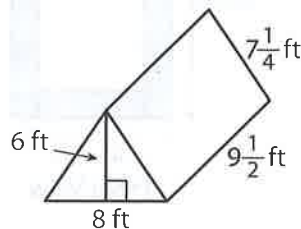
In this project you will perform a famous probability experiment called "Buffon's Needle." It will enable you to calculate π to a high degree of accuracy.

- Choose a long, thin, straight, rigid item for the "needle" such as a toothpick, a piece of uncooked spaghetti, or an unsharpened pencil. Since you will be throwing your "needle" many times, you can use many of them, but they must be identical.
- Draw or make a set of long, narrow, parallel lines. This is your "target." The lines must be the same distance apart as the length of your "needle."
- Toss or drop your needle so that it lands on the target. If the needle is intersecting one of the parallel lines when it comes to rest, record the toss as an "intersection."
- Repeat *at least* 200 times. The more times you toss your needle, the closer to π your results will be.
- Evaluate $\frac{\text{number of tosses}}{\text{number of intersections}} \times 2$ for your approximation of π .
- Create a presentation describing your experiment in detail. Be sure to explain how you created your target and any problems you may have had. Use the space below to write down any questions you have or important information from your teacher.



MATH IN CAREERS ACTIVITY

Product Design Engineer Miranda is a product design engineer working for a sporting goods company. She designs a tent in the shape of a triangular prism. The approximate dimensions of the tent are shown in the diagram. How many square feet of material does Miranda need to make the tent (including the floor)? What is the volume of the tent? Show your work.



Study Guide Review

MODULE **10**

Analyzing and Comparing Data

Key Vocabulary

mean absolute deviation (MAD) (*desviación absoluta media, (DAM)*)

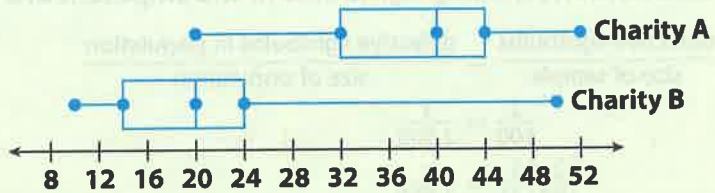


ESSENTIAL QUESTION

How can you solve problems by analyzing and comparing data?

EXAMPLE

The box plots show amounts donated to two charities at a fundraising drive. Compare the shapes, centers and spreads of the box plots.



Shapes: The lengths of the boxes and overall plot lengths are fairly similar, but while the whiskers for Charity A are similar in length, Charity B has a very short whisker and a very long whisker.

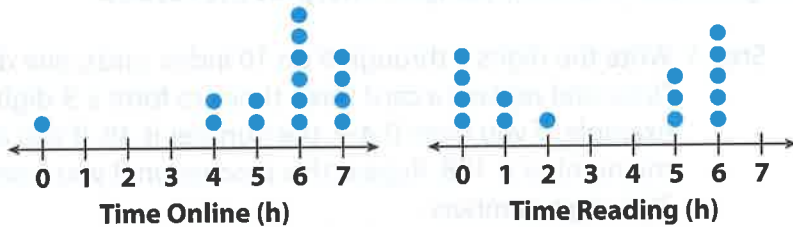
Centers: The median for Charity A is \$40, and for Charity B is \$20.

Spreads: The interquartile range for Charity A is $44 - 32 = 12$. The interquartile range for Charity B is slightly less, $24 - 14 = 10$.

The donations varied more for Charity B and were lower overall.

EXERCISES

The dot plots show the number of hours a group of students spends online each week, and how many hours they spend reading. (*Lesson 10.1*)



- Calculate the medians of the dot plots. _____
- The average times (in minutes) a group of students spends studying and watching TV per school day are given. (*Lesson 10.3*)

Studying: 25, 30, 35, 45, 60, 60, 70, 75
 Watching TV: 0, 35, 35, 45, 50, 50, 70, 75

- Find the mean times for studying and for watching TV.

- Find the mean absolute deviations (MADs) for each data set.

- Find the difference of the means as a multiple of the MAD. _____

Random Samples and Populations



ESSENTIAL QUESTION

How can you use random samples and populations to solve real-world problems?

Key Vocabulary

biased sample (*muestra sesgada*)

population (*población*)

random sample (*muestra aleatoria*)

sample (*muestra*)

EXAMPLE 1

An engineer at a lightbulb factory chooses a random sample of 100 lightbulbs from a shipment of 2,500 and finds that 2 of them are defective. How many lightbulbs in the shipment are likely to be defective?

$$\frac{\text{defective lightbulbs}}{\text{size of sample}} = \frac{\text{defective lightbulbs in population}}{\text{size of population}}$$

$$\frac{2}{100} = \frac{x}{2,500}$$

$$\frac{2 \cdot 25}{100 \cdot 25} = \frac{x}{2,500}$$

$$x = 50$$

In a shipment of 2,500 lightbulbs, 50 are likely to be defective.

EXAMPLE 2

The 300 students in a school are about to vote for student body president. There are two candidates, Jay and Serena, and each candidate has about the same amount of support. Use a simulation to generate a random sample. Interpret the results.

Step 1: Write the digits 0 through 9 on 10 index cards, one digit per card. Draw and replace a card three times to form a 3-digit number. For example, if you draw 0-4-9, the number is 49. If you draw 1-0-8, the number is 108. Repeat this process until you have a sample of 30 3-digit numbers.

Step 2: Let the numbers from 1 to 150 represent votes for Jay and the numbers from 151 to 300 represent votes for Serena. For example:

Jay: 83, 37, 16, 4, 127, 93, 9, 62, 91, 75, 13, 35, 94, 26, 60, 120, 36, 73

Serena: 217, 292, 252, 186, 296, 218, 284, 278, 209, 296, 190, 300

Step 3: Notice that 18 of the 30 numbers represent votes for Jay. The results suggest that Jay will receive $\frac{18}{30} = 60\%$ of the 300 votes, or 180 votes.

Step 4: Based on this one sample, Jay will win the election. The results of samples can vary. Repeating the simulation many times and looking at the pattern across the different samples will produce more reliable results.

EXERCISES

1. Molly uses the school directory to select, at random, 25 students from her school for a survey on which sports people like to watch on television. She calls the students and asks them, "Do you think basketball is the best sport to watch on television?" (Lesson 11.1)

- a. Did Molly survey a random sample or a biased sample of the students at her school?

- b. Was the question she asked an unbiased question? Explain your answer.

2. There are 2,300 licensed dogs in Clarkson. A random sample of 50 of the dogs in Clarkson shows that 8 have ID microchips implanted. How many dogs in Clarkson are likely to have ID microchips implanted? (Lesson 11.2)

3. A store gets a shipment of 500 MP3 players. Twenty-five of the players are defective, and the rest are working. A graphing calculator is used to generate 20 random numbers to simulate a random sample of the players. (Lesson 11.3)

A list of 20 randomly generated numbers representing MP3 players is:

474	77	101	156	378	188	116	458	230	333
78	19	67	5	191	124	226	496	481	161

- a. Let numbers 1 to 25 represent players that are _____.
- b. Let numbers 21 to 500 represent players that are _____.
- c. How many players in this sample are expected to be defective? _____
- d. If 300 players are chosen at random from the shipment, how many are expected to be defective based on the sample? Does the sample provide a reasonable inference? Explain.

Unit Project

COMMON CORE

7.RP.2c, 7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4

A Sample? Simple!

For this project, choose one of the following topics. Randomly sample at least 25 people and record their answers to the question you write about the topic.

- Number of pets in a home
- Number of books read optionally in the past 6 months
- Number of cell phone lines a family uses
- Number of full-time or part-time students in a family
- Number of hours of sleep obtained *last night*

Use your data to create a presentation that includes the following:

- An explanation of how you chose your random sample
- The question you asked and the answers you received
- A box plot of your data
- Your interpretations of the data, including the median, the range, and the most common items of data
- Your inference, based on your data, of the number of 5,000 randomly chosen people who would give the answer to your question that your median group gave

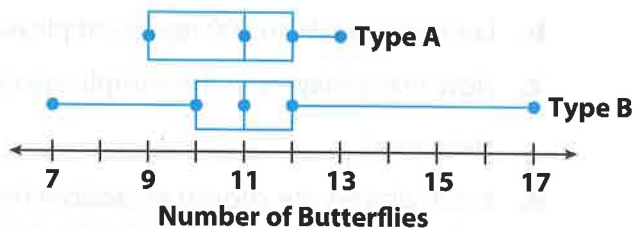
Use the space below to write down any questions you have or important information from your teacher.



MATH IN CAREERS ACTIVITY

Entomologist An entomologist is studying how two different types of flowers appeal to butterflies. The box-and-whisker plots show the number of butterflies that visited one of two different types of flowers in a field. The data were collected

over a two-week period, for one hour each day. Find the median, range, and interquartile range for each data set. If you had to choose one flower as having the more consistent visits, which would you choose? Explain your reasoning.



MODULE 12 Experimental Probability



ESSENTIAL QUESTION

How can you use experimental probability to solve real-world problems?

EXAMPLE 1

What is the probability of picking a red marble from a jar with 5 green marbles and 2 red marbles?

$$P(\text{picking a red marble}) = \frac{\text{number of red marbles}}{\text{number of total marbles}}$$

$$= \frac{2}{7} \quad \begin{array}{l} \text{There are 2 red marbles.} \\ \text{The total number of marbles is } 2 + 5 = 7. \end{array}$$

EXAMPLE 2

For one month, a doctor recorded information about new patients as shown in the table.

	Senior	Adult	Young adult	Child
Female	5	8	2	14
Male	3	10	1	17

What is the experimental probability that his next new patient is a female adult?

$$P(\text{new patient is a female adult}) = \frac{\text{number of female adults}}{\text{total number of patients}}$$

$$P = \frac{8}{60} = \frac{2}{15}$$

What is the experimental probability that his next new patient is a child?

$$P(\text{new patient is a child}) = \frac{\text{number of children}}{\text{total number of patients}}$$

$$P = \frac{31}{60}$$

EXERCISES

Find the probability of each event. (Lesson 12.1)

1. Rolling a 5 on a fair number cube.

3. Picking a blue marble from a bag of 4 red marbles, 6 blue marbles, and 1 white marble.

2. Picking a 7 from a standard deck of 52 cards. A standard deck includes 4 cards of each number from 2 to 10.

4. Rolling a number greater than 7 on a 12-sided number cube.

Key Vocabulary

- complement (*complemento*)
- compound event (*suceso compuesto*)
- event (*suceso*)
- experiment (*experimento*)
- experimental probability (*probabilidad experimental*)
- outcome (*resultado*)
- probability (*probabilidad*)
- sample space (*espacio muestral*)
- simple event (*suceso simple*)
- simulation (*simulación*)
- trial (*prueba*)

5. Christopher picked coins randomly from his piggy bank and got the numbers of coins shown in the table. Find each experimental probability. (Lessons 12.2, 12.3)

Penny	Nickel	Dime	Quarter
7	2	8	6

- a. The next coin that Christopher picks is a quarter. _____
- b. The next coin that Christopher picks is not a quarter. _____
- c. The next coin that Christopher picks is a penny or a nickel. _____
6. A grocery store manager found that 54% of customers usually bring their own bags. In one afternoon, 82 out of 124 customers brought their own grocery bags. Did a greater or lesser number of people than usual bring their own bags? (Lesson 12.4)

MODULE 13

Theoretical Probability



ESSENTIAL QUESTION

How can you use theoretical probability to solve real-world problems?

Key Vocabulary

theoretical probability
(probabilidad teórica)

EXAMPLE 1

- A. Lola rolls two fair number cubes. What is the probability that the two numbers Lola rolls include at least one 4 and have a product of at least 16?

There are 5 pairs of numbers that include a 4 and have a product of at least 16:

(4, 4), (4, 5), (4, 6), (5, 4), (6, 4)

Find the probability.

$$P = \frac{\text{number of possible ways}}{\text{total number of possible outcomes}} = \frac{5}{36}$$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

- B. Suppose Lola rolls the two number cubes 180 times. Predict how many times she will roll two numbers that include a pair of numbers like the ones described above.

One way to answer is to write and solve an equation.

$$\frac{5}{36} \times 180 = x \quad \text{Multiply the probability by the total number of rolls.}$$

$$25 = x \quad \text{Solve for } x.$$

Lola can expect to roll two numbers that include at least one 4 and have a product of 16 or more about 25 times.

EXAMPLE 2

A store has a sale bin of soup cans. There are 6 cans of chicken noodle soup, 8 cans of split pea soup, 8 cans of minestrone, and 13 cans of vegetable soup. Find the probability of picking each type of soup at random. Then predict what kind of soup a customer is most likely to pick.

$$P(\text{chicken noodle}) = \frac{6}{35}$$

$$P(\text{split pea}) = \frac{8}{35}$$

$$P(\text{minestrone}) = \frac{8}{35}$$

$$P(\text{vegetable}) = \frac{13}{35}$$

The customer is most likely to pick vegetable soup. That is the event that has the greatest probability.

EXERCISES

Find the probability of each event. (Lessons 13.1, 13.2)

1. Graciela picks a white mouse at random from a bin of 8 white mice, 2 gray mice, and 2 brown mice.
2. Theo spins a spinner that has 12 equal sections marked 1 through 12. It does **not** land on 1.

3. Patty tosses a coin and rolls a number cube. (Lesson 13.3)

- a. Find the probability that the coin lands on heads and the cube lands on an even number.

- b. Patty tosses the coin and rolls the number cube 60 times. Predict how many times the coin will land on heads and the cube will land on an even number.

4. Rajan's school is having a raffle. The school sold raffle tickets with 3-digit numbers. Each digit is either 1, 2, or 3. The school also sold 2 tickets with the number 000. Which number is more likely to be picked, 123 or 000? (Lesson 13.3)

5. Suppose you know that over the last 10 years, the probability that your town would have at least one major storm was 40%. Describe a simulation that you could use to find the experimental probability that your town will have at least one major storm in at least 3 of the next 5 years. (Lesson 13.4)

A Birthday Puzzle

Here's a fact that you may find hard to believe: In any group of 23 randomly chosen people, it is more likely than unlikely that two of them share the same birthday!

To test the rule, collect information for a list of 23 people and their birthdays. The people on your list should be randomly chosen from a well-defined group, such as 23 members of a major league baseball team, the first 23 governors of your state, 23 Olympic swimming champions, or 23 Nobel Prize winners in chemistry. Create a presentation showing the names and birthdays. Tell whether any two of your people share the same birthday and explain how you determined that fact. Then share your results with the other students in your class. Determine how many of them had results that supported the "23" rule and how many did not.



Use the space below to write down any questions you have or important information from your teacher.

MATH IN CAREERS ACTIVITY

Meteorologist A meteorologist predicts a 20% chance of rain for the next two nights and a 75% chance of rain on the third night.

Tara would like to go camping for the next 3 nights but will not go if it is likely to rain on all 3 nights. Should she go? Use probability to justify your answer.

Study Guide Review

MODULE 14 Real Numbers



ESSENTIAL QUESTION

How can you use real numbers to solve real-world problems?

EXAMPLE 1

Write $0.\overline{81}$ as a fraction in simplest form.

$$x = 0.\overline{81}$$

$$(100)x = (100)0.\overline{81}$$

$$100x = 81.\overline{81}$$

$$-x = -0.\overline{81}$$

$$99x = 81$$

$$x = \frac{81}{99}$$

$$x = \frac{9}{11}$$

100 times $0.\overline{81}$ is $81.\overline{81}$.

Divide both sides by 99.

Simplify.

EXAMPLE 2

Solve each equation for x .

A $x^2 = 289$

$$x = \pm\sqrt{289}$$

$$x = \pm 17$$

The solutions are 17 and -17 .

B $x^3 = 1000$

$$x = \sqrt[3]{1000}$$

$$x = 10$$

The solution is 10.

EXAMPLE 3

Write all names that apply to each number.

A $5.\overline{4}$
rational, real

$5.\overline{4}$ is a repeating decimal.

B $\frac{8}{4}$
whole, integer, rational, real

$$\frac{8}{4} = 2$$

C $\sqrt{13}$
irrational, real

13 is a whole number that is not a perfect square.

Key Vocabulary

cube root (*raíz cúbica*)

irrational number (*número irracional*)

perfect cube (*cuvo perfecto*)

perfect square (*cuadrado perfecto*)

principal square root (*raíz cuadrada principal*)

rational number (*número racional*)

real number (*número real*)

repeating decimal (*decimal periódico*)

square root (*raíz cuadrada*)

terminating decimal
(*decimal finito*)

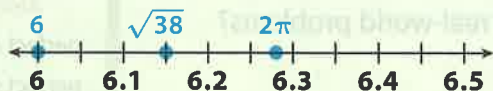
EXAMPLE 4

Order 6, 2π , and $\sqrt{38}$ from least to greatest.

2π is approximately equal to 2×3.14 , or 6.28.

$\sqrt{38}$ is approximately 6.15 based on the following reasoning.

$$\sqrt{36} < \sqrt{38} < \sqrt{49} \quad 6 < \sqrt{38} < 7 \quad 6.1^2 = 37.21 \quad 6.2^2 = 38.44$$



From least to greatest, the numbers are 6, $\sqrt{38}$, and 2π .

EXERCISES

Find the two square roots of each number. If the number is not a perfect square, approximate the values to one decimal place.

(Lesson 14.1)

1. 16 _____

2. $\frac{4}{25}$ _____

3. 225 _____

4. $\frac{1}{49}$ _____

5. $\sqrt{10}$ _____

6. $\sqrt{18}$ _____

Write each decimal as a fraction in simplest form. (Lesson 14.1)

7. $0.\bar{5}$ _____

8. $0.\bar{63}$ _____

9. $0.\bar{214}$ _____

Solve each equation for x . (Lesson 14.1)

10. $x^2 = 361$

11. $x^3 = 1728$

12. $x^2 = \frac{49}{121}$

Write all names that apply to each number. (Lesson 14.2)

13. $\frac{2}{3}$

14. $-\sqrt{100}$

15. $\frac{15}{5}$

16. $\sqrt{21}$

Compare. Write $<$, $>$, or $=$. (Lesson 14.3)

17. $\sqrt{7} + 5$ $7 + \sqrt{5}$

18. $6 + \sqrt{8}$ $\sqrt{6} + 8$

19. $\sqrt{4} - 2$ $4 - \sqrt{2}$

Order the numbers from least to greatest. (Lesson 14.3)

20. $\sqrt{81}$, $\frac{72}{7}$, 8.9 _____

21. $\sqrt{7}$, 2.55, $\frac{7}{3}$ _____

Exponents and Scientific Notation

Key Vocabulary

scientific notation

(notación científica)



ESSENTIAL QUESTION

How can you use scientific notation to solve real-world problems?

EXAMPLE 1

Write each measurement in scientific notation.

- A** The diameter of Earth at the equator is approximately 12,700 kilometers.

Move the decimal point in 12,700 four places to the left: 1.2700.

$$12,700 = 1.27 \times 10^4$$

- B** The diameter of a human hair is approximately 0.00254 centimeters.

Move the decimal point in 0.00254 three places to the right: 0.00254.

$$0.00254 = 2.54 \times 10^{-3}$$

EXAMPLE 2

Find the quotient: $\frac{2.4 \times 10^7}{9.6 \times 10^3}$.

Divide the multipliers: $2.4 \div 9.6 = 0.25$

Divide the powers of ten: $\frac{10^7}{10^3} = 10^{7-3} = 10^4$

Combine the answers and write the product in scientific notation.

$$0.25 \times 10^4 = 0.25 \times (10 \times 10^3) = (0.25 \times 10) \times 10^3 = 2.5 \times 10^3$$

EXERCISES

Write each number in scientific notation. (Lessons 15.2, 15.3)

1. 25,500,000 _____ 2. 0.00734 _____

Write each number in standard notation. (Lessons 15.2, 15.3)

3. 5.23×10^4 _____ 4. 1.33×10^{-5} _____

Simplify each expression. (Lessons 15.1, 15.4)

5. $(9 - 7)^3 \cdot 5^0 + (8 + 3)^2$ _____ 6. $\frac{(4 + 2)^2}{[(9 - 3)^3]^2}$ _____

7. $3.2 \times 10^5 + 1.25 \times 10^4 + 2.9 \times 10^5$ _____ 8. $(2600)(3.24 \times 10^4)$ _____

The Large and the Small of It

Did you know that, according to one estimate, there are about 300,000,000,000 birds in the world? Or that the mass of a dust particle is about 0.0000000008 kg?

For this project, find five interesting facts that involve numbers greater than one million. Find five more facts that involve positive numbers less than one-millionth. Then create a presentation that includes the following:

- Each of the ten numbers written in both standard notation and scientific notation
- A description of the fact that each number represents
- The source where you found the information
- An image for each fact



Use the space below to write down any questions you have or important information from your teacher.

MATH IN CAREERS ACTIVITY

Astronomer An astronomer is studying Mars, which is the closest planet to our Sun. Mars is 57,910,000,000 meters away from the Sun. Earth is 149,600,000,000 meters away from the Sun. How much longer does it take for light to travel from the Sun to Earth than to Mars? The speed of light is 3.0×10^8 miles per second.